Companion Appendix for *"Dynamic Adjustment of Fiscal Policy under a Debt Crisis"*

(not for publication)

September 17, 2017

Abstract

In this Companion Appendix we provide numerical examples to our theoretical results and detailed derivations. Note that this is not a section for robustness of the results as our results are analytically prooved. It is a section that provides numerical examples to craft better the papers' message and, in turn, it can be used as a companion for the reader of the main draft.

JEL classification: *E6;H6;H30.*

Keywords: Fiscal sustainability; Fiscal rules; Bond-financed deficits

Acknowledgments: We have benefited from comments and suggestions by C. Azariadis, H. Dellas, M. Froemel, J.C. Martinez, A. Philippopoulos, C. Rapti, E. Vella, R. Wendner and participants at various conferences and seminars. Part of the project was conducted during my visit in Brown University, whose hospitality is gratefully acknowledged. The usual disclaimer applies.

1 Further Numerical Examples to the Analytical Results

In this section we provide numerical examples to our theoretical results. Note that this is not a section for robustness of the results of as all of our results are analytically prooved. It is a section that provides numerical examples to craft better the papers' message, so it is used a companion for the reader of the main draft.

1.1 Countries that have the same level of debt but differences in initial capital stock

This numerical example aims to convey the message that the current level of debt alone is not a sufficient condition for the stability of an equilibrium.

Following our numerical example in the main draft lets assume now that Country B has lower debt than before and equal to the debt of Country , $B_0 = 0.2$ (the same as of Country A). For illustation lets call this Country C. Country A and C differ only in their initial capital stock, $K_0^A = 0.5$ and $K_0^C = 3$.

The dynamics are the following. Exactly as in the main draft, if both countries follow the same rule, $b = 0.013$ and $a = 0.5$, then, Country A debt will follow a sustainable path (Figure A1.1) and Countrys' C debt will explode (Figure A2.2). While if $a = 0.9$ then Country C will stabilize its debt, (Table A1.3). The dynamics are the same as those provided in the main draft.

Figure A1.1

Country A: Dynamic adjustment towards the stable steady-state

with $a = 0.5$ and $b = 0.013$

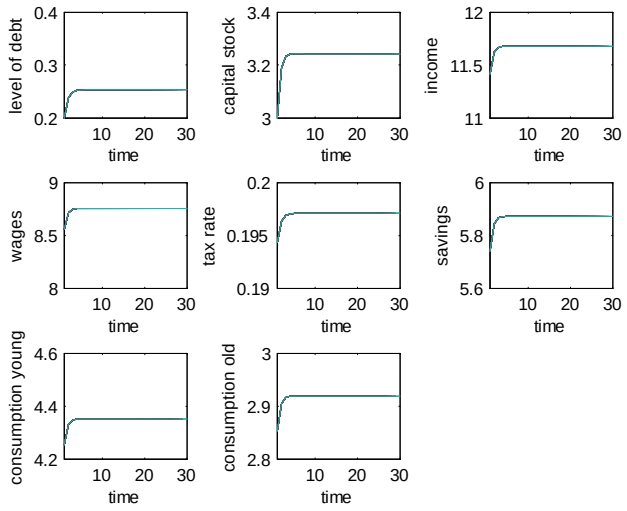


Figure A1.2

Country C: Dynamic adjustment towards exploding debt

with $a = 0.5$ and $b = 0.013$

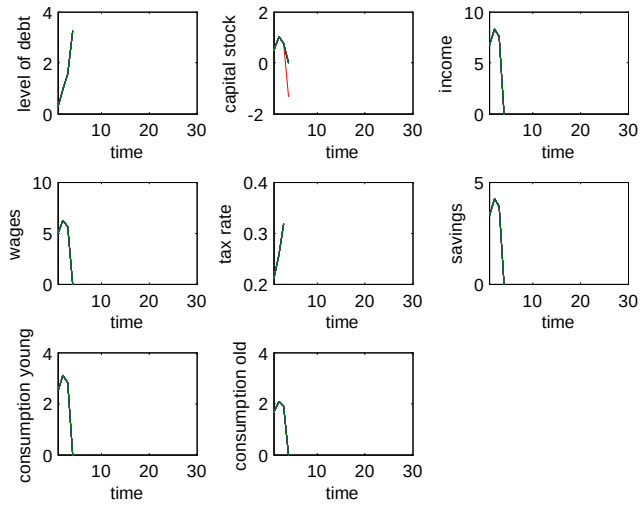
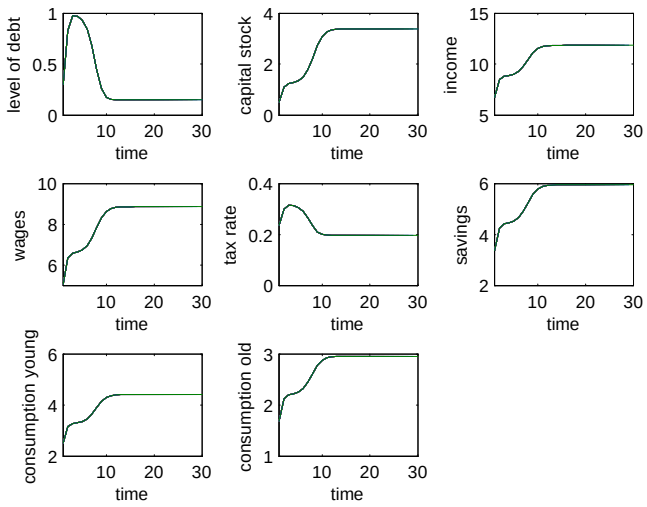


Figure A1.3

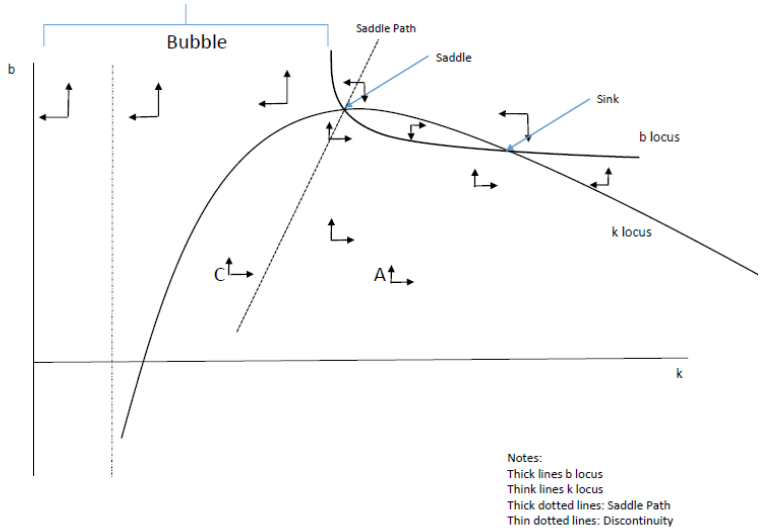
Country C: Dynamic adjustment towards the stable steady-state

with $a = 0.9$ and $b = 0.013$



As I theoretical prooved those examples rely on the following phase diagram which indicates the position of both countries (same B but different K) when they both follow the same rule:

Figure A1.4



The message of this example is that the current level of debt alone is not a sufficient condition for the stability of an equilibrium but government should take into account the state of cycle (capital stock) of the economys' output along with the parameters of the fiscal policy rule.

1.2 Countries that have the different level of debt but the same initial capital stock

In this subsection we provide the example where countries are at the same level of development but they face different level of initial level of debt. This example conveys the message that the current level of capital stock alone is not a sufficient condition for the stable path for the debt.

Following the same structural parameter values in the main draft lets assume two countries where both have initial level of capital stock $K_0 = 1$ and both follow the same rule, $a = 0.5$ and $b = 0.013$. However, Country D has initial level of debt $B_0 = 0.3$ while Country E has initial level of debt, $B_0 = 0.5$ (for a graphical illustration on the phase plane see Figure A2.4 below). As can be seen from the following figures country E debt is at a sustainable path (Figure A2.1) while country D debt explodes (Figure A2.2). If the level of debt is sufficiently high, then, higher austerity has to be placed ($a = 0.8$) in order Country E to overcome its unsustainable dynamics (Figure A2.3.1) and taxation follows a non-mononotic path towards debt stabilization.

Figure A2.1

Country E: Dynamic adjustment towards the stable steady-state

with $a = 0.5$ and $b = 0.013$

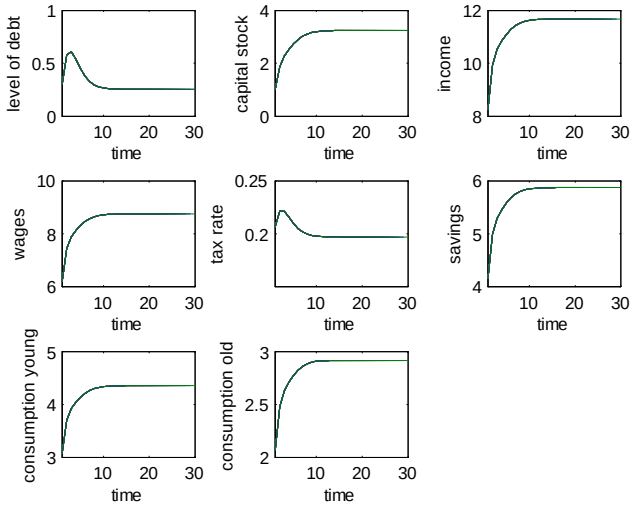


Figure A2.2

Country D: Dynamic adjustment towards exploding debt

with $a = 0.5$ and $b = 0.013$

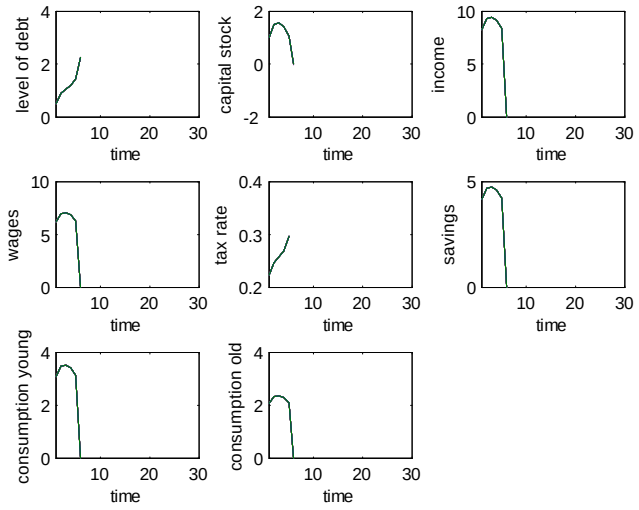
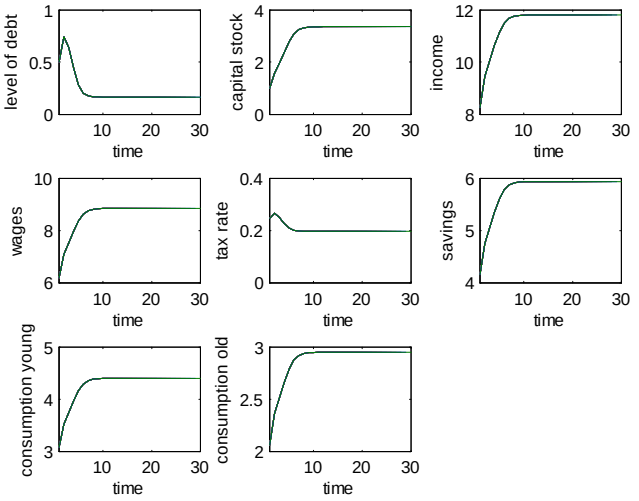


Figure A2.3.1

Country D: Dynamic adjustment towards the stable steady-state

with $a = 0.8$ and $b = 0.013$



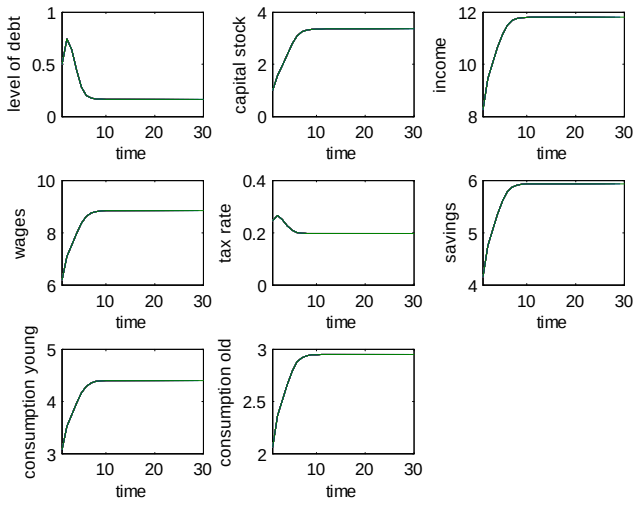
Last, in the following graph we show that if additional fiscal discipline is implemented, $a = 1.1$ then because in Country D the initial capital stock is sufficiently high, the adjustment dynamics of taxation can be monotonically decreasing. This happens as for high initial capital

stock, the relative marginal productivity of private investment is high, thus, lower taxation is necessary to boost private investment under high fiscal consolidation.

Figure A2.3.2

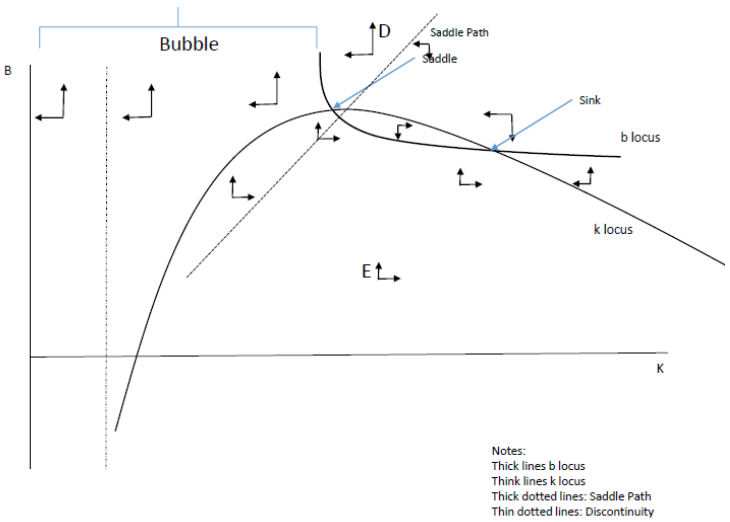
Country D: Dynamic adjustment towards the stable steady-state

with $a = 1.1$ and $b = 0.013$



As I theoretical prooved those examples rely on the phase diagram that describes those cases is their initial state. When a increases the k-locus shifts upwards (see Figure 2 in the text) and county D dynamics alter as it is now, in the sustainability area.

Figure A2.4



2 Extending the Model with Endogenous Labour Supply

2.1 The model and its results

We consider an overlapping generations model as in our main draft. There are N_t consumers who each live for two periods. They choose their consumption today, C_t, and tomorrow, d_{t+1}, and leisure, l_t, to maximize intertemporal utility as given by the following utility function,

$$U = \ln C_t + \beta \ln d_{t+1} + \chi \ln l_t \tag{1}$$

where $\beta \in (0,1)$ is the weight that agents place in their second period utility and $\chi \in (0,1)$ is the weight that agents place on leisure satisfaction. In the first period of their life, agents allocate their unit labour time to receive a salary, $w_t(1-l_t)$ which is taxed by τ_t. When old, the agents consume their savings and they receive a return on their savings, r_{t+1}. The intertemporal budget constraint is given by

$$C_t + \frac{d_{t+1}}{1+r_{t+1}} = w_t(1-l_t)(1-\tau_t)$$

Then, from the first order conditions, $\frac{1}{C_t} - \lambda = 0$, $\frac{\beta}{d_{t+1}} - \frac{\lambda}{1+r_t} = 0$ and $\frac{\chi}{l_t} - \lambda w_t(1-\tau_t) = 0$ we obtain:

$$C_t = \frac{1}{1+\beta+\chi} w_t(1-\tau_t) \tag{2}$$

$$d_{t+1} = \frac{\beta(1+r_{t+1})}{1+\beta+\chi} w_t(1-\tau_t) \tag{3}$$

$$l_t = \frac{\chi}{1+\beta+\chi} \tag{4}$$

$$S = \tilde{s}(1-\tau_t)w_t \tag{5}$$

where now the savings propensity $\tilde{s} \equiv \frac{\beta}{1+\beta+\chi}$ depends on time preference parameter and the preference for leisure. The preference for leisure, χ, negatively affects savings prospensity and in turn, the first order effect of taxation on savings (i.e. keeping wages constant) lowers as the preference for leisure increases.

On the supply side, there exists a continuum of firms that produces output, Y_t, using capital, k_t, labour, n_t, and a public good supplied by the government g_t,

$$Y_t = Ak_t^{\alpha} n_t^{1-\alpha} g_t^{\gamma} \qquad \alpha+\gamma<1 \tag{6}$$

The wage rate and return on capital, using the labour market clearing condition, $n_t + l_t = 1$, are determined by

$$w_t = (1-\alpha)A\left(\frac{1+\beta}{1+\beta+\chi}\right)^{-\alpha} k_t^{\alpha} g_t^{\gamma} \tag{7}$$

$$R_t = \alpha A\left(\frac{1+\beta}{1+\beta+\chi}\right)^{1-\alpha} k_t^{\alpha-1} g_t^{\gamma} \tag{8}$$

Using equation (6) of our main draft we get $w_t = (1-\alpha)\tilde{A}(\frac{1+\beta}{1+\beta+\chi})^{-\alpha}k_t^{\alpha}$ and $R_t = \alpha\tilde{A}(\frac{1+\beta}{1+\beta+\chi})^{1-\alpha}k_t^{\alpha-1}$ and defining $\hat{A} \equiv A(\frac{1+\beta}{1+\beta+\chi})^{1-\alpha}$ then we get

$$w_t = (1-\alpha)\hat{A}\left(\frac{1+\beta+\chi}{1+\beta}\right)k_t^{\alpha} \tag{9}$$

$$R_t = \alpha\hat{A}k_t^{\alpha-1} \tag{10}$$

where those equations function are qualilatively similar to our main model. Then, using (9) into (5) we get $S(w_t) = \frac{\beta}{1+\beta+\chi}(1-\tau_t)w_t = \frac{\beta}{1+\beta+\chi}(1-\tau_t)w_t = \frac{\beta}{1+\beta+\chi}(1-\tau_t)(1-\alpha)\hat{A}\left(\frac{1+\beta+\chi}{1+\beta}\right)k_t^{\alpha}g_t^{\gamma}$

$$S(w_t) = (1-\alpha)(1-\tau_t)\hat{A}k_t^{\alpha}$$

We can obtain through the process in our main Appendix 1, the augmented dynamical system with endogenous labour supply as

$$k_{t+1} - k_t = (\tilde{s}(1-\alpha) + \tilde{s}(b-\gamma) - b)\, y(k_t) - k_t + (a(1-s) - R_t(k_t))\, B_t \tag{11}$$

$$B_{t+1} - B_t = (R(k_t) - a - 1)B_t + by(k_t) \tag{12}$$

where differently to the original framework, the leisure preference parameter affects the savings propensity $\tilde{s}$ and factor productivity $\tilde{A}$ (through (8) and $y(k_t)$). As can be easily seen the properties of equilibrium and, in turn, our qualitative results do not depend on leisure choise parameters. The main change is that $\tilde{s}$ and $\hat{A}$ now have to satisfy the correspoding conditions of $\tilde{s}$ and $\tilde{A}$ where for $\chi \in (0,1)$ are satisfied. Interestingly, the leisure choice parameter affect the thresholds for sustainability (in the oposite way β does) and the policy rule parameters as it affects the responsiveness of individual on taxation. However, our system has the same properties as displayed graphically in Figure 1 of the main draft.

2.2 Dynamic Analysis of the Model with Endogenous Labour Supply

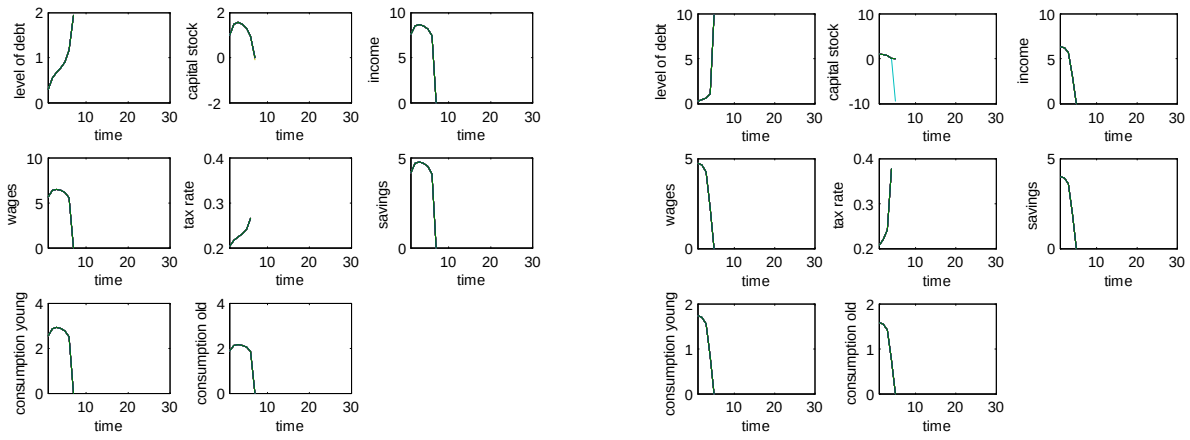
In this subsection, we provide a numerical example of our model extended with endogenous labour supply. Athough as modelled above endogenous leisure does not affect the qualitative dynamics and results of our main framework (the Phase Diagram above remains the same) it can be a useful tool for studying the thresholds of the policy rule parameters that can drive the economy to the sustainability area. The analytical proof of the following numerical example is straightforward from the Appendix of the main draft of the paper.

According to the theoretical model above, the higher the preference for leisure, χ the lower the savings's prospensity of individuals, $\frac{\partial \tilde{s}}{\partial \chi} < 0$. The higher χ, the less responsive of savings from changes in taxes for a given wage rate, $\frac{\partial S}{\partial \tau} = -\tilde{s} w_t$ (i.e. higher,χ, lower, $\tilde{s}$, lower $\frac{\partial S}{\partial \tau}$). Also the higher χ, the higher the leisure choise and the higher labour quantity,n. Those mechanism will affect quantitatively the threshold for the police rule regarding debt stabilization, a. From on hand, the higher χ the lower the amount of labour, the lower the tax base the higher the need for taxation for the provision of government expenses to boost the economy to overcome

the recession, thus the higher a. Also, once for higher χ the lower the savings prospensity, higher responsiveness of debt to taxes while distort at a lower level private savings allowing for higher a. Thus, one would expect the if χ is low, the lower a has to be to stabilize the economy, while the when χ is high, the higher a has to be.

We provide a numerical example to the above statement. Lets assume a country, Country D, with initial capital stock $K_0 = 1$ and level of debt $b = 0.3$. In Case 1, we assume that the preference for leisure, $\chi = 0.2$ and in Case 2, $\chi = 0.7$. Then, following the policy rule parameters, $a = 0.4$ and $b = 0.013$ (we just use a slighly lower a to provide more ilustrative graphs) both economies debt explods:

Figure A3.1: $a = 0.4$

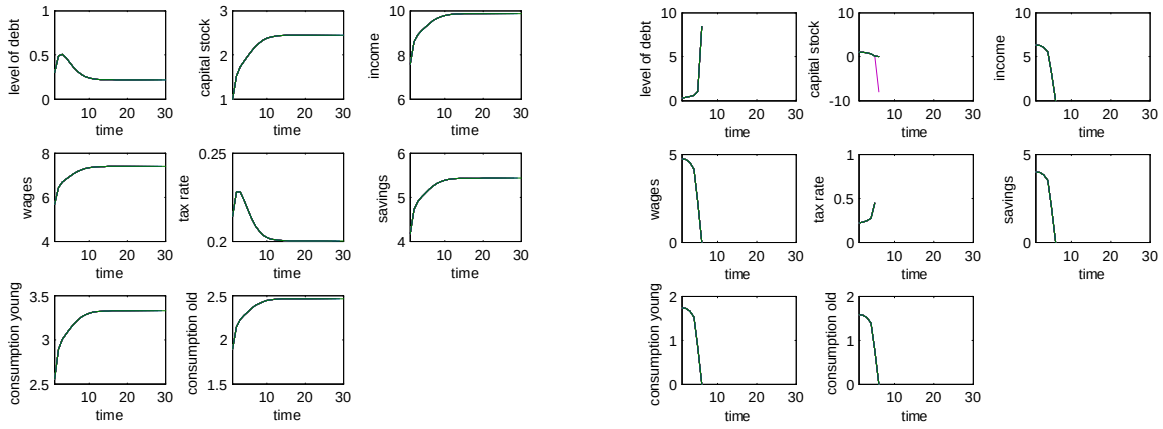


Left Panel Case 1: $\chi = 0.2$ Right PaneL Case 2: $\chi = 0.7$

According to Figure A3.1 for any level of leisure preference, for a low level of fiscal austerity the economys' debt explodes as in the case of Country B in our main text.

Figure A3.2. Dynamic adjustment towards exploding vs non-expoding debt according to the preference for leisuret

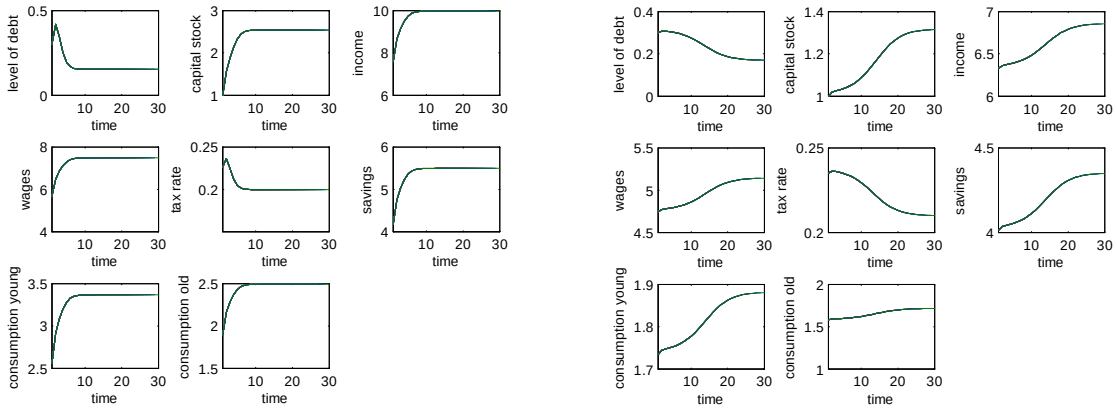
with $a = 0.6$



Left Panel Case 1: $\chi = 0.2$ Right Panel Case 2: $\chi = 0.7$

According to Figure A3.2 for higher responsiveness of the tax rate on debt, a from 0.4 to 0.6 (government increases taxes more for any increase in debt to reduce the primary deficit) the, in the case where $\chi = 0.2$ the economy stabilizes the debt while in the case where $\chi = 0.7$ (low responsivness of savings to taxation) the economy's debt still explodes. This is in line of what we analyzed before.

Figure A3.3. Dynamic adjustment towards exploding vs non-expoding debt according to the preference for leisure
with $a = 0.82$



Left Panel Case 1: $\chi = 0.2$ Right Panel Case 2: $\chi = 0.7$

According to Figure A3.3 for higher responsiveness of the tax rate on debt, a from 0.6 to 0.82 then also the economy of Case 2, Right Panel can stabilize its debt.

To sum up, endogenous labour supply plays an important role for the threshold of sustainability of the policy rule regarding fiscal consolidation. The higher the preference for leisure the higher the level of fiscal austerity necessary so as the economy to stabilize its debt. Intuition of this result is analyzed in the main text of the paper as well.