Aging, Growth and the Allocation of Public Expenditures on Health and Education

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Abstract

In this paper, we develop an overlapping generations endogenous growth model in which both public education and health are sources of growth by affecting the accumulation rate of the human capital stock and the savings rate over life expectancy. We first find that dynamic complementarities of public expenditures lead to minimum threshold levels of public education and health expenditures that ensure sustainable growth. Considering endogenous fertility, we then study the process of aging and its effect on endogenous government policy. We show how governments can use the allocation of public expenditures as an alternative policy instrument to maximize growth without increasing the tax rate or the retirement age as usually happens in aging economies.

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1 Introduction

The demographic transition of the last decades has critically affected governments budgets and raised concerns for sustainable economic growth. The decline in both mortality and fertility rates has led to an aging population[1] and to increasing pressure for public spending on health. In turn, aging has created concerns about an increase in the retirement age or an increase in the tax rate to finance higher health expenditures (Shelton, 2008).

From a positive view, many studies examined the impact of changes in longevity and fertility on economic growth. In particular, the literature on longevity and growth has studied the effect of mortality decline on fertility choices and on the return of private investment in education (Easterlin, 1996; De-La.Croix and Licandro, 1999; Kalemni-Oscan et al., 2000; Soares, 2006). Other studies have shown how the mortality rate jointly affects fertility choice, savings, private education, and economic growth (Blackburn and Cipriani, 2002; Zhang and Zhang, 2005). Although demographic changes affect the stucture of the population, individual choises and the tax base of the economy, little work has been done on the endogenous responce of government policy and the associated effects on endogenous economic growth.[2]

In this paper, we investigate the long-run growth properties and fiscal policy implications of aging in the form of increasing longevity and decreasing fertility. We show how the government can use the allocation of existing government revenues in health and education under the financial pressure of population aging instead of increasing the tax rate or increasing the retirement age. The mechanism that drives our results is that increasing longevity not only affects the fertility rate in the economy and the return of private investment in education, as in, among others, Zhang et al. (2003), Doepke (2004), Zhang and Zhang (2005), Soares (2006), Hazan and Zoabi (2006), but it also endogenously affects the tax base of the economy and the relative return of public health and education investment on economic growth.

Several empirical facts have motivated our analysis. Developing countries tend to be charac-

[1] For a review of the relationship between declining mortality rates and subsequent fertility reductions during the demographic transition, see, among others, Easterlin (1996) (Chap. 6).

[2] In a seminal paper on exogenous longevity and endogenous fertility choice, Zhang et al. (2003) studied the effect of mortality decline not only on savings, growth and investment in education but also on the endogenous tax rate determined by a median voter mechanism.

terized by a positive relation between improvements in health and economic growth.[3] Growth stagnation is often linked to a lack of sufficient public policies on health and education (Rivera and Currais, 1999; World-Bank, 2003). To this end, in many countries public expenditures on education and health constitute the greatest part of social spending, which is constantly increasing.[4] Furthermore, a closer inspection of the data shows that the growth impact of health capital decreases at relatively large endowments of health stock (Schultz, 1997; Kwabena and Wilson, 2004). Intuitively, increased spending on health can negatively affect the growth rate of the economy because it crowds out other public activities, such as investment in education, for which the relative return in terms of growth can increase.[5]

Motivated by the aforementioned stylized facts, we build an Overlapping Generations (OLG) endogenous growth model that relates to the literature on human capital formation, demographics, health, public policy and economic growth. Human capital formation builds upon the works of Glomm and Ravikumar (1992), Glomm and Ravikumar (1997), Blankenau and Simpson (2004), Blankenau et al. (2007) and Yew and Zhang (2009), in which public investment in human capital, the allocation of agents' time on education increase the quality (productivity) of the labor force and, thus, the growth rate of the economy. Additionally, following Chakraborty

[3]Empirical time series analyses by Arora (2001) and Schultz (1997) have provided evidence of a strong relationship between health, productivity and growth. In addition, Lorentzen et al. (2008) focus on many channels through which death affects growth. Also, there is much debate currently about whether health improvements enhance growth via productivity or capital accumulation, or whether they retard growth by increasing population size and diluted capital per worker. Recent work includes Acemoglu and Johnson (2007) and Weil (2007)

[4]For instance, public expenditures accounted for 84.5% of total health expenditure in the United Kingdom, 83.3% in Sweden and 77.4% in Japan(Gomez et al., 2001). A number of countries in 2000 had public education shares close to 100%, such as Norway (98.7%), Turkey and Portugal (98.6% each), Finland (98%) and Sweden (97%)(Osang and Sarkar, 2007). Of the 36 OECD and non-OECD countries covered in this study, 19 countries (53%) financed at least 90% of their overall educational expenditures through public spending in 2000(Osang and Sarkar, 2007).

[5]In a similar vein, Zhang and Zhang (2005) and Soares (2005, 2006) found that a decline in population mortality and fertility rates increases the return of private investment in education. Also, AJ find a negative effect of health on income per capita attributed to a Malthusian effect of higher population growth on income. We complement this literature by investigating not only the effect of longevity on fertility (quantity of population), but also the impact of longevity on the relative return of public expenditures on health and education (relative quality of public expenditures).

(2004) and Bhattacharya and Qiao (2007), we assume that public health expenditures positively affect life expectancy. In this setup, we also embed the analyses of Zhang et al. (2001), Zhang et al. (2003) and Zhang and Zhang (2005) on the effect of changes in longevity on fertility choice, private education and savings. Then, in a unified framework, we examine the subsequent feedback effects of aging on the endogenous allocation of public health and education expenditures and the tax rate, focusing, following Blankenau and Simpson (2004), on long-run growth.[6]

Our major findings can be summarized as follows. First, we find that threshold levels of government taxation and public expenditures for health and education are necessary for the existence of a balanced growth path. In particular, the government should provide a minimum level of public health expenditure to ensure that the effects of public education expenditures on productivity are not eliminated by the low level of life expectancy. Second, we find that the shares of health and education expenditures in total public spending are crucial for the design of long-run fiscal policies under demographic changes. Rising longevity affects the relative return of investing in health and education and, at the same time, declining fertility affects the population structure and thus the tax base of the economy. We find that due to the change in relative returns of health and education on growth, the government can use the allocation of existing public revenues to improve the long-run growth rate of the economy, without necessarily increasing the tax rate.

Some aspects of our work should be stressed in comparison with the existing literature on demographics and public policy. First, we show how the allocation of existing revenues between health and education is important for the emergence of growth stagnation expanding the set of mechanisms through the complementary relation of public expenditures on growth. Second, we provide a unified framework that stresses the importance of the endogenous composition of productive government expenditures (among others Barro, 1990; Devarajan et al., 1996; Barro and Sala-i-Martin, 1992) under the effect of demographic changes on public health, human capital, fertility, longevity and in turn, on economic growth (Doepke, 2004; Chakraborty, 2004;

[6] Blankenau and Simpson (2004) study the effect of public education expenditures on economic growth, under the long-run growth maximizing objective. These authors study the role of tax structure and technology parameters on this relation. In our paper, we also consider the role of demographic parameters and the endogenous composition of government expenditures between health and education.

Zhang and Zhang, 2005; Hazan and Zoabi, 2006; Soares, 2006; Zhang et al., 2006). In particular, literature on demographics and public policy study the effect of demographic parameters and exogenous policy instruments, usually the tax rates, on the growth rate in the economy. In this paper, we also show how demographic parameters affect endogenously policy instruments under the growth maximizing objective. To this end, we derive optimal government responce, setting growth maximizing policy rules, to demographic shifts not only by choosing the tax rate but also the internal allocation of governement revenues. In our framework, the Barro (1990) taxation rule is suboptimal and depends not only on production technology (as in Blankenau and Simpson, 2004) but also on demographic parameters through the composition of public spending. We establish that when the population ages, savings increase, making public investment in education more productive (relative to health) in terms of growth due to higher physical capital stock that complements human capital in the production. Also, the health status in the economy exhibits decreasing returns to improvements in health due to limits imposed by the natural death rate. We show that as the relative return of public investment in education increases, the government can change the allocation of existing revenues for education and increase the growth rate and output (tax base). In turn, a lower tax rate will be required for financing public expenditures on health.

The rest of the paper is structured as follows. Section 2 establishes and solves the optimization problem of households and firms. Section 3 presents the competitive decentralized equilibrium and studies the existence and properties of the equilibrium growth rate. Section 4, investigates the growth-maximizing fiscal policies. Finally, Section 5 concludes the paper and discusses further research directions.

2 The model

This section presents an OLG model that builds upon Glomm and Ravikumar (1997), Blankenau and Simpson (2004), Chakraborty (2004) and Zhang and Zhang (2005). The main features of the model are as follows: (a) the agents have finite and uncertain life-time horizons, (b) they are endowed with a given amount of time allocated to work, rearing their children and providing for their education, (c) the productivity of labor depends on education through time allocation by parents and public expenditures on education, (d) public health expenditures affect the health stock in the economy that in turn, positively affects life expectancy.

2.1 Demand side

Consider an economy with overlapping generations of identical N agents that face a finite lifetime horizon and consume a single good. Agents learn when young and work in middle age with a certain lifetime horizon.[7] Old agents face a probability of being alive at the retirement period, given by $\phi(m_t)$, which depends on the health stock in the economy, m_t, and has the following properties:

Assumption 1 $\phi(m_t) \in [0,1]$ *is continuous*, $\phi(0) = 0$, $\lim\limits_{m \to \infty} \phi(m_t) = 1$.

Assumption 2 $\phi^{'} \equiv \frac{\partial \phi(m_t)}{\partial m_t} > 0$.

Assumption 3 $\phi^{''} \equiv \frac{\partial^2 \phi(m_t)}{\partial (m_t)^2} \leq 0$.

Assumption 1 comes from the definition of a probability function.[8] Assumptions 2 and 3 posit that the health stock in the economy affects positively the probability of being alive in the retirement period at a non-increasing rate, following, among others, Chakraborty (2004).[9]

Each working agent, i, is endowed with one unit of time, supplies labor, L_{it}, rears children and provides for their education. Following Becker et al. (1990) and Zhang and Zhang (2005) we assume that rearing a child requires $\sigma > 0$ constant units of time. In turn, the agent's labor income, I_L, in the working period is given by:

$$I_L \equiv w_t h_t (1 - \sigma n_t - e_t n_t) \tag{1}$$

[7]Following Zhang et al. (2001) and Zhang and Zhang (2005), we assume a fixed working period. We focus on other instruments to mitigate the financing pressure in aging economies rather than the standard increase (reduction) in working (retirement) period.

[8]The zero lower bound of the probability function is used for analytical tractability. Our results are robust with the use of a non-zero lower bound and are available upon request.

[9]A non-convex production function implies that the effect of health on life expectancy is higher for low health endowments (Kwabena and Wilson, 2004; Zhang and Zhang, 2005) and is more consistent with the upper bound limit of the probability function.

where $w_t h_t$ denotes the quality adjusted wage rate, n_t denotes the number of children at period t and e_t denotes time allocation for the education of children.

Because the lifetime horizon after retirement is uncertain, we assume (similarly, among others, to Zhang and Zhang, 2005) the existence of an actuarially fair annuity market, which maps savings to investment in physical capital, K, for production in the next period. In particular, the agents save part of their labor income to annuity assets, s_t, and, assuming that the private insurance market is competitive, they obtain a return for a unit of saving equal to r^A_{t+1} where r^A denotes the return on annuity assets. In the event of being alive, the agent's income in the retirement period, I_R, is given by[10]:

$$I_R \equiv (1 + r^A_{t+1})s_t \tag{2}$$

Following Glomm (1997), Galor and Weil (2000), and Kalemni-Oscan (2003), the objective of the agent is to maximize intertemporal utility, given by

$$U(c_t, c_{o,t+1}, n_t) = \ln c_t + \rho\phi \ln c_{o,t+1} + \eta \ln n_t w_{t+1} h_{t+1} \tag{3}$$

where c_t denotes consumption in the working period, t, $c_{o,t+1}$ denotes consumption in the retirement period and h_{t+1} denotes the human capital of children. Parameters $\rho \in (0, 1)$ and $\eta > 0$ correspond to the rate of time preference and the preference for an additional child, respectively.[11] The third term of the utility function indicates that parents are imperfectly altruistic towards their offsprings. Specifically, parents obtain satisfaction from the income of their offsprings (Galor and Weil, 2000). This is meant to capture the idea that parents care about their offsprings future prospects and social status (both being enhanced through more advanced knowledge and/or increased income).

Following Glomm and Ravikumar (1992), Glomm and Kaganovich (2008) and Yew and Zhang (2009), the human capital production of individuals depends positively on parents' allocation of time, on the human capital stock of parents, h_t, and on average human capital expenditures provided by the government, $\bar{H}^E = \frac{H^E}{N_t}$. It is given by

$$h_{t+1} = v(e_t h_t)^\mu (\bar{H}^E)^{1-\mu} \tag{4}$$

[10]Following Zhang and Zhang (2005) and others, a fair life insurance company redistributes wealth of people who died among those who survive. As will be shown later on, actuarial fairness requires that $(1 + r^A) = \frac{(1+r)}{\phi}$ where r denotes the rate of return on capital.

[11]See Zhang et al. (2001), Zhang and Zhang (2005) for a similar specification of the utility function.

where $v > 0$ denotes an exogenous parameter for the production of human capital and $\mu \in (0,1)$ is a parameter that measures the intensity of private and public provision in the accumulation of human capital.

2.2 Production side

Regarding the supply side, we assume that is produced an homogenous good and the aggregate production depends on physical capital, K_t and effective labour $h_t L_t$, given by

$$Y_t = K_t^{\alpha}(h_t L_t)^{1-\alpha} \tag{5}$$

where $0 < \alpha < 1$ denotes the share of physical capital in the production function, Y_t denotes total output, and L_t denotes the number of workers. Labor productivity depends on human capital obtained in childhood.[12] Firms operate under a competitive environment and take factor prices as given. The first-order conditions of the after-tax profits maximization problem of firms are then given by

$$r_t = (1-\tau)\alpha K_t^{\alpha-1}(h_t L_t)^{1-\alpha} \tag{6}$$

$$w_t = (1-\tau)(1-\alpha)K_t^{\alpha}(h_t L_t)^{-\alpha} \tag{7}$$

where equations (6) and (7) state that the marginal productivity of capital and the marginal productivity of effective labor have to equal their factor prices. Also, τ denotes a flat tax rate on output.

2.3 Government

The presence of public goods justifies policy intervention in this model. In particular, the government spends on health and education to enhance the accumulation of human capital and the health. The accumulation rate of human capital is affected by the public sector size by a

[12] In a previous version of the paper, a more general production function where labour productivity, A, depended both on learning-by-investing externaties (without scale effects) and human capital, $A = \left(\left(\frac{K_t}{L_t}\right)^{\beta}(H_t)^{1-\beta}\right)$, provided an additional channel were aging through savings and capital externalities affected the productivity of labour. Our results are robust to this specification. An interesting research direction is to consider the effect of aging on gross investment and productivity and, in turn, on growth maximizing policies with and without scale effects.

flat tax rate on output and by altering the allocation of public expenditures between education and health.

More specifically, we assume that the government levies a flat tax rate, τ, where $0 < \tau \leq 1$, on income to finance government revenues through a balanced budget given by

$$H_t^E + H_t^M = \tau Y_t \tag{8}$$

Government revenues, τY_t, are used for the provision of public education and health expenditures, given by H_t^E and H_t^M, respectively. To ease exposition, we parameterize public health expenditures, H_t^M, as a share of tax revenues, denoted by $\psi \in [0, 1]$. The internal allocation of government expenditures can then be written as

$$H_t^E = (1 - \psi)\tau Y_t \tag{9}$$

$$H_t^M = \psi \tau Y_t. \tag{10}$$

Hence, in addition to τ, ψ comprises a policy instrument of the government.

The government by providing health expenditures positively affects the health status in the economy given by

$$m_t = \xi \frac{H_t^M}{Y_t} \tag{11}$$

The equation (11) shows that the social health status of the economy is affected positively by the share of income devoted to health, H_t^M / Y_t, where $\xi > 0$ denotes the technology that is applied to investment in health. The justification behind (11) stems from the strong correlation between public health as a share of output and health quality, as found by Barro (1996), Schultz (1997), Rivera and Currais (1999), and Osang and Sarkar (2007). Intuitively, the formulation of health status by (11) implies that output has a positive effect on health through the provision of health expenditures and a negative effect through activities that are products of output, such as pollution and obesity.

2.4 The Household's Problem

The agent's problem is to choose c_t, n_t, e_t, s_t to maximize intertemporal utility (3) subject to the resource constraints given by

$$w_t h_t(1 - \sigma n_t - e_t n_t) = c_t + s_t \tag{12}$$

and

$$c_{o,t+1} = \frac{(1 + r_{t+1})}{\phi} s_t. \tag{13}$$

Equation (12) states that agents allocate their labor income to consumption and savings, $I_L = c_t + s_t$, and equation (13) states that the fraction of retired persons who are alive consume income from the savings of their working period.

The interior solution of the above maximization problem is given by the following optimal allocations:

$$n_t^* = \frac{\eta\mu}{\left[1 + \phi(m_t)\rho + \eta\right] e_t^*} \tag{14}$$

$$s_t^* = \frac{\phi(m_t)\rho}{1 + \phi(m_t)\rho + \eta} w_t h_t \tag{15}$$

$$e_t^* = \frac{\mu\sigma}{(1 - \mu)}, \tag{16}$$

where n_t^* denotes the optimal number of children that parents have to rear. Equation (14) shows that the optimal number of children is positively related to their taste for children in the utility function, η, negatively related to the time allocation to education, e, and negatively related to the probability of being alive in the retirement period, ϕ. These effects reflect the quantity-quality trade-off regarding children faced by the middle-aged agents and show that the rate of fertility in absolute terms falls with the decrease in the rate of mortality, thus triggering population aging.[13]

Equation (15) determines the optimal allocation for asset holdings as a function of the wage rate and the model parameters. This equation shows that the savings of the economy depend linearly and positively on the wage rate, negatively on the taste for children in the utility function and positively on the length of life. An increase in the survival probability positively

[13] These results coincide with empirical facts on the demographic transition in the last century in which a fall in the mortality rate has been accompanied by a fall in the fertility rate (see, among others Lorentzen et al., 2008; Easterlin, 1996).

affects savings because agents store more capital for their greater lifetime. Finally, equation (16) shows that time to education increases with the intensity of private education in the human capital production function and with the cost of per child rearing time.

3 The Competitive Decentralized Equilibrium

We can now define the competitive decentralized equilibrium in our model and present the solution for the aggregate economy and its equilibrium dynamics.

3.1 Definition of equilibrium and market clearing

Definition 1 *The competitive decentralized equilibrium (CDE) of the economy is defined for the exogenous policy instruments τ and ψ, factor prices r_t and w_t, such that:*

i) Individuals solve their intertemporal utility maximization problem by choosing c_t, s_t, n_t, and e_t.

ii) Firms choose effective labour and capital to maximize their profits.

iii) All markets clear. The market clearing condition for the capital market is given by

$$K_{t+1} = s_t N_t \tag{17}$$

The market clearing condition for the labour market is given by

$$L_t = (1 - \sigma n_t - e_t n_t) N_t. \tag{18}$$

iv) The government budget constraint holds.

Equation (17) shows that in equilibrium net investment in capital has to equal aggregate savings, which are determined by the aggregate purchase of annuity assets. [14]

3.2 Competitive Equilibrium and Balanced Growth Path

Using the equilibrium conditions for a symmetric equilibrium, $k_t = \frac{K_t}{N_t}$, combining the optimal wage rate, (7), on the optimal savings rate (15), plugging the optimal time allocation of

[14] As standard in OLG models, equation (17) incorporates the assumption that its period (ussualy approximated to 30 years) the physical capital stock fully depreciates.

education, (16), into the optimal number of children, (14), and substituting into the capital and labour market clearing condition, (17), we obtain the accumulation rate of the per capita physical capital stock in equilibrium, g_k, as follows:

$$g_k \equiv \frac{k_{t+1} - k_t}{k_t} = \Psi(\psi, \tau) \left(\frac{h_t}{k_t} \right)^{(1-\alpha)} - 1 \tag{19}$$

where $\Psi(\psi, \tau; \rho, \eta, \xi, \alpha, \sigma) \equiv \frac{\phi(m_t)\rho(1-\alpha)(1-\tau)(\phi(m_t)\rho+1)^{-\alpha}\sigma}{(1+\phi(m_t)\rho+\eta)^{-\alpha}(1-\mu)\eta}$ is a function of parameters and policy instruments and from (11) and (10) it follows that $\phi(m_t) = \phi(\xi\psi\tau)$.

Substituting the allocation of public extenditure on education, (9), and the optimal allocation of agents on education, (16), in (4), and using the equilibrium conditions and the production technology, we obtain the accumulation of the human capital stock, g_h, as follows:

$$g_h \equiv \frac{h_{t+1} - h_t}{h_t} = \upsilon\left((1-\psi)\tau\right)^{1-\mu} \left(\frac{\mu\sigma}{(1-\mu)} \right)^{\mu} \left(\frac{\phi(\xi\psi\tau)\rho + 1}{(1 + \phi(\xi\psi\tau)\rho + \eta)} \right)^{(1-\alpha)(1-\mu)} \left(\frac{h_t}{k_t} \right)^{-\alpha(1-\mu)} - 1 \tag{20}$$

Equations (19) and (20) characterize the dynamics of the competitive equilibrium which in turn determine the equilibrium growth rate.

The Balanced Growth Path (BGP) is defined as a state in which all the variables of the economy grow at a constant rate. In this economy, at the BGP, the aggregate equilibrium growth rates of human and private capital have to grow at the same rate, i.e., $\frac{h_{t+1}-h_t}{h_t} = \frac{k_{t+1}-k_t}{k_t} = g$. This result is easily obtained by investigating the equilibrium growth rates of these variables separately. From (20) we find that for the human capital stock to grow at a constant rate, this growth rate has to equal the growth rate of the private capital stock to eliminate diminishing returns to capital. This condition also satisfies equation (19). Thus, a necessary condition for a balanced growth path in our economy is that $g_H = g_K = g$.

Given the above result for the BGP we can now derive the equilibrium growth rate of the economy. Rearranging (19) we find that at the BGP for the long-run ratio of human to physical capital stock, $\tilde{z} = h/k$, is given by

$$\tilde{z} = \left(\frac{(\tilde{g} + 1)}{\Psi(\tau, \psi)} \right)^{\frac{1}{1-a}} \tag{21}$$

. Then, the (21) in (20), we can then obtain the long-run growth rate, $\tilde{g}$, in our economy as

$$\Phi(\tilde{g}) \equiv \tilde{g} - v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{\tilde{g}+1}{\Psi(\tau,\psi)}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}}+1 \tag{22}$$

where the solution of this continuous function for $\tilde{g} > 0$, such that $\Phi(\tilde{g}) = 0$, determines the existence and the properties of the equilibrium long-run growth rate, $\tilde{g}$, given by the following proposition[15]:

Proposition 1 *There exists a unique and strictly positive long-run equilibrium growth rate per capita, $\tilde{g} > 0$, iff $v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{1}{\Psi(\tau,\psi)}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1$ for any parameter values in their assumed domain and some values of the policy instruments, and it is given by the fixed point in (22), $\tilde{g} : \Phi(\tilde{g}) = 0$.*

Proof. *See Appendix 1.* ▪

Proposition 1 states the necessary and sufficient condition for the existence of positive long-run growth rate. The saving rate of the economy and public expenditures on health and education has to be sufficient enough so as to ensure positive growth. Notice that our economy will exhibit a zero equilibrium growth rate if the necessary and sufficient parametric condition of Proposition 1 holds with equality. A crucial remark is that the policy instruments, τ and ψ, are critical not only for the quantitative determination of the equilibrium growth rate but also for the existence of a strictly positive growth rate in the BGP. In fact, a direct consequence of Proposition 1 is that there exists a subset in the domain of the policy instruments that forms a set of sufficient values for sustainable growth. The following corollaries formalize this point.

Corollary 1 *Given the parametric characteristics of the economy, there exists a range $(\check{\psi}, \hat{\psi}) \in (0,1)$ of public health expenditures as a share of total public expenditures that has to be implemented for Proposition 1 to hold. The maximum, $\hat{\psi}$, and minimum, $\check{\psi}$, threshold levels of the health expenditures share in total public spending are given by*

$$v\left((1-x)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi x\tau)\rho+1}{(1+\phi(\xi x\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{1}{\Psi(\tau,x)}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1 \text{ where } x = \check{\psi}, \hat{\psi} \tag{23}$$

[15] In the Companion Appendix, we conduct dynamic analysis and we show that the long-run growth rate is stable. Also, the dynamics of the equilibrium growth rate and the ratio of human to physical capital stock towars the steady-state are monotonic. Once the equilibrium is stable, the comparative statics analysis of Section 4 is meaningful.

Proof. See Appendix 2. ▪

Corollary 1 establishes the threshold levels of public expenditures on health and education for the economy to exhibit endogenous growth with a positive long-run growth rate. These threshold levels arise from the complementarities in the growth process between education expenditures on human capital accumulation and health expenditures on the saving rate (through the survival probability). Intuitively, an increase in investment in education will raise human capital accumulation and output. If health expenditures remain constant, their share in the total output will fall, and by (11), less health stock will be formed. The lower level of health stock will lead to lower life expectancy, lower saving rate and lower tax base and, in turn, to a decrease in human capital accumulation. At low health stock levels, the increase in human capital accumulation will be absorbed by the decrease in life expectancy, and thus the economy will exhibit zero or negative growth unless a minimum amount of public health expenditures takes place. This result is consistent with empirical evidence by Baldacci et al. (2008) and Rivera and Currais (1999), in which a lack of sufficient health expenditures reduces the favorable growth effects of education in developing countries through low school enrollments, disabilities and epidemic diseases.

A by-product of our analysis is that a similar methodology applies for the lower and the upper bound of the government size required to ensure positive growth. It is straightforward to show that Proposition 1 imposes boundaries for the tax rate (because it does not hold for $\tau = 0$ or $\tau = 1$) that are determined endogenously by the structural characteristics of the economy. Thus, a change in the population structure or the technological characteristics of the economy can expand or restrict the range of values of these policy instruments. The following Corollary to Proposition 1 determines the boundaries for the government size.

Corollary 2 *Given the parametric characteristics of the economy, there exists a range of values for the tax rate on income,* $\underline{\tau} < \tau < \bar{\tau}$ *, that has to be implemented by the government for Proposition 1 to hold. The maximum,* $\bar{\tau}$*, and minimum,* $\underline{\tau}$*, threshold levels of the government size are given by:*

$$v\left((1-\psi)j\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi j)\rho+1}{(1+\phi(\xi\psi j)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{1}{\Psi(j,\psi)}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1 \textit{ where } j = \bar{\tau}, \underline{\tau} \tag{24}$$

Proof. *See Appendix 3.* ▪

Corollary 2 sets the upper and lower bound for the government size. Sustainable growth cannot be attained if the government size exceeds $\bar{\tau}$ because of the high distortion to private savings and low physical capital accumulation. In addition, the tax rate has to be greater than $\underline{\tau}$ for the government to finance expenditures on health and education that drive growth in this economy. These maximum threshold levels depend on population aging determined by a positive change in the probability of living in the retirement period, ϕ. A rise in fertility can generate fiscal limitations (lower tax base in the next period because of the lower labor force), while population aging can expand the range of feasible taxation on output depending on the actual level of the tax rate and the composition of public expenditures.

Our findings seem to conform to the view that development traps can be the result of the lack of sufficient health and education expenditures and the associated government size that guarantee positive growth. In the following section, by endogenizing government policy aiming at growth-maximization, we will show how government can use the composition of public expenditures and government size not only to preserve positive growth but also to enhance growth under demographic changes.

4 Growth-maximizing policies

In this section, we will analyze growth-maximizing fiscal policy rules to assess the impact of taxation and the contribution of the components of public expenditures on long-run growth. Modern growth theory has shown particular interest in growth-enhancing policies, as the understanding of the forces of economic growth is crucial in order to identify the relative merits and synergies of government interventions. Moreover, the growth rate is usually the main measurable objective of the government. Although earlier papers, like Barro (1990), have mostly considered welfare and growth-maximizing policies under a unified perspective, subsequent studies, among others Blankenau and Simpson (2004), have emphasized the role of growth maximization as an independent policy target.

4.1 Growth-maximizing policy rules

Definition 2 *Growth-maximizing policies in the competitive equilibrium of the aggregate economy are given under Definition 1 when the government acts as a Stackelberg player and chooses* τ^* *and* ψ^* *to maximize the long-run growth rate of the economy by taking into account the aggregate maximizing behavior of the competitive equilibrium, and when the government budget constraints, feasibility and technological conditions are met.*

The problem of long-run growth-maximizing policies can be formulated as

$$\max_{\tau,\psi} \tilde{g}(\tau,\psi) = \Psi(\tau,\psi)(\tilde{z})^{(1-\alpha)} - 1 \tag{25}$$

where $\tilde{z} \equiv z(\tau,\psi)$ is implicitly (using equation (19) and (20)) given by the decentralized equilibrium response of the private agents:

$$\Psi(\tau,\psi)\,(\tilde{z})^{(1-\alpha)} - \upsilon\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}(\tilde{z})^{-\alpha(1-\mu)} = 0 \tag{26}$$

The solution and properties of this problem are given by the following proposition:

Proposition 2 *The growth maximizing tax rate,* τ^**, and share of total public expenditures to health,* ψ^**, under endogenous longevity to public health expenditures is given by the following system of equations*

$$\tau^* = \frac{1-a}{1-a\psi^*} \tag{27}$$

$$\phi(\xi\psi^*\tau^*)^{-1}\acute{\phi}\xi + \frac{(1-2\alpha)}{\alpha}\frac{\rho\eta\xi\acute{\phi}}{(\phi(\xi\psi^*\tau^*)\rho+1)(1+\phi(\xi\psi^*\tau^*)\rho+\eta)} = \frac{(1-\alpha)}{\alpha\tau^*(1-\psi^*)} \tag{28}$$

The growth maximizing tax rate is higher than the Barro (1990), τ^b*, optimal taxation rule,* $\tau^* > 1-a = \tau^b$*, and depends positively on health expenditures as a share of total government revenues,* $\frac{\partial\tau^*}{\partial\psi^*} > 0$.

Proof. *See Appendix 4.* ∎

Futhermore, the growth-maximizing physical to human capital stock is given by

$$z^* = \left(\frac{v \left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu} ((1-\psi^*)\tau^*)^{1-\mu}}{\phi(\xi\psi^*\tau^*) \left(\frac{\rho(1-\alpha)(1-\tau^*)\sigma}{(1-\mu)\eta}\right) \left(\frac{(\phi(\xi\psi^*\tau^*)\rho+1)}{(1+\phi(\xi\psi^*\tau^*)\rho+\eta)}\right)^{\mu(1-\alpha)-1}} \right)^{\frac{1}{1-\alpha\mu}} \tag{29}$$

Equations (27), (28), and (29) express the three endogenous variables, τ^*, ψ^*, z^*, in terms of the model parameters (v, α, μ, ξ, ρ) and determine the rules that the government has to satisfy to attain growth-maximization.

In detail, equation (27) represents the growth-maximizing tax rate, which is positively affected by public health expenditures. This tax rate is higher than the elasticity of the aggregate human capital stock in the production function because health expenditures raise the benefit of taxation relative to the standard endogenous growth models. The Barro (1990) optimal taxation rule, which states that the optimal tax rate is equal to the elasticity of the public capital stock in the production function, ($\tau^b = 1 - a$), is suboptimal here unless health expenditures as a share of total public spending are equal to zero, $\psi = 0$. This occurs because a rise in the tax rate increases human capital accumulation and output through the elasticity of aggregate human capital in the production function, but at the same time, under the endogeneity of longevity, health expenditures have to increase at the BGP. Thus, the marginal cost of public funds increases. In turn, the government size has to exceed the elasticity of aggregate human capital stock in the production function for the marginal cost of public funds to be equal to the corresponding marginal benefit through the increase in life expectancy.

Equation (28) determines the growth-maximizing allocation rule for public expenditures. This allocation rule states that at the margin, health expenditures should have the same effect on the growth rate of the economy through the change in the survival probability as the marginal expenditure spent on education through the change in human capital accumulation (equalizing relative returns on growth).

4.2 The effect of population aging on growth-maximizing policies

In this section we numerically solve the nonlinear system (27), (28), and (29). In particular, we study the effect of a parameter value that triggers population aging through our framework on the growth-maximizing tax rate and the allocation of government expenditures. We assume as a source of population aging a positive change in the health technology parameter, ξ.[16] Concerning parameterization, the values of the parameters are in line with those in the literature or are chosen to yield plausible values for the fertility rate and survival probability.

In particular, for the probability of surviving, we use a concave function, $\phi(m) = \frac{m}{1+m}$, that satisfies the Assumptions 1-3.[17] The value for the share of physical capital in the production function, $\alpha = 0.3$, comes from many studies in the empirics of growth theory as in Mankiw et al. (1992). Following common practice, we use the productivity parameter in human capital accumulation, v, as a scale parameter to help us get plausible values for output growth rate, around 2%. We also set the cost of rearing children, $\sigma = 0.1$, to achieve time allocation to labour $(1 - \sigma n_t - e_t n_t)$ around, 70% (labour force participation). Also, we set the preference for an additional child, $\eta = 0.6$ so as to yield fertility rate, n, around 1.5. Last, we set time preference rate to $\rho = 0.95$ so as to calibrate the net saving rate to output ratio around 6% (gross savings to output ratio for the US on 2011 in the World Bank Dataset is 11%). Last, we set the initial parameter ξ to 6 to calibrate health expenditures as a share of total government spending around 20% as it is given in the World Health Organization Database.

In Table 1 we report the numerical results of the growth rate, the fertility rate, the probability of surviving and the endogenous policy parameters for varying values of parameter ξ in range, $6 < \xi < 7$ by grid 0.2.[18] Our qualitative results show that an increase in the technology of the health stock accumulation leads to an increase in life expectancy and a decline in the fertility rate, thus, triggering population aging. In turn, the government shifts the allocation of expenditures from health to education, leading to an increase in the growth rate. The higher growth rate implies a higher tax base and results in a decrease in the growth-maximizing tax rate.

Intuitively, an increase in the survival probability during the retirement period (population aging because the fertility rate decreases) has two effects on private agents' decisions. First, agents in the working period reduce the time of rearing children and increase work to increase consumption in the longer retirement period (see (14)). Second, they overaccumulate private

[16] For the role of such changes on health parameters in causing population aging, see Aisa and Sanso (2006).

[17] Our numerical results are very robust to many other specifications of the probability function and are available upon request.

[18] In the Companion Appendix, we provide an extended Table with grid 0.2 and other variables that are by product of the main variables of the model such as labour force participation and savings to output ratio.

capital through higher savings (see (15)) for their greater lifetime. The overaccumulation of private capital is suboptimal because agents do not internalize the favorable effects of public expenditures through higher growth, and, in turn, raises the growth-maximizing tax rate ("taxation" effect). This effect is reinforced by the fact that fewer children form a lower tax base in the next period. However, at the same time, population aging affects the relative return of investing in health and education. Aging decreases the relative return of investing in health as the probability of surviving is upward bounded to increases in heath (due to the natural death rate). Also, the higher accumulation of physical capital stock by the rise in savings increases the productivity of labor through the production function. Because of this, by reallocating expenditures from health to education, the government can enhance the tax base of the economy through higher growth and, following (27), reduce the government size. In other words, the additional tax revenues finance public education at a greater proportion ("re-allocation" effect) and reduce the need for higher taxes.

A direct policy implication of the above analysis is that countries facing fiscal limitations can re-allocate the existing public resources by steering expenditures towards (away from) education and away from (towards) health in response to a decline (rise) in the mortality rate. The numerical results presented above, show that as the population ages, countries should target low tax rates accompanied by a higher share of public expenditure on education.

5 Concluding remarks

According to the aim of the paper, we explored the steady state and fiscal policy implications of public health and education expenditures in an endogenous growth model under the presence of population aging driven by a decline in mortality and fertility rate. We showed that a minimum amount of public expenditures has to be devoted to health and education for the economy to attain positive long-run growth. This result conforms to the empirical findings provided by Rivera and Currais (1999) and Baldacci et al. (2008), who have argued that a minimum level of health and education provision for the labor force is necessary to maintain continuous growth.

We also found that the allocation of public expenditures between health and education is a critical determinant of the taxation rule. The share of health expenditures to total public expenditures positively affects the optimal government size due to the change in the marginal cost of public funds. A fall in the mortality rate not only affects fertility and the tax base of the economy but also causes a change in the relative return of public investment in health and education on the growth rate. Our results indicate that the policy change in the efficient allocation of resources induced by a decline in the mortality rate in the retirement period counteracts the positive effect on the tax rate, because the return on health and education on growth changes. Hence, in the presence of population aging, the government can now benefit from the optimal re-allocation of public expenditures between education and health. The efficient composition of public expenditures becomes more important in the presence of binding financial constraints such as those posed by the International Monetary Fund and the World Bank to less developed economies, which limit the provision of expenditures for social services.

We believe that this framework and the endogenous policy mechanism provide many additional research directions. A straightforward extension is to consider different social security regimes, and mainly a pay-as-you go system (as in Zhang et al., 2001). A pay-as-you go system would provide an additional channel for the fertility choice of agents in the model and the efficiency of the tax base. Futhermore, the negative effect of income on the health stock in the economy can be explicitly modelled by introducing the law of motion for pollution or the obesity level in the economy as positive functions of income. Such an extension would enrich our analysis by addressing other issues, such as policies for environmental sustainability and reduction in obesity rates. Lastly, welfare maximization policies can also be considered, although in this OLG setup, political economy issues arise regarding the generation that will be better off.

6 Appendix

Appendix 1. *Proof of Proposition 1*

The properties of $\Phi(\tilde{g})=\tilde{g}-v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{\tilde{g}+1}{\Psi(\tau,\psi)}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}}+1$ are given as follows:

1. Continuous in $\tilde{g}$ from the addition of continuous functions.
2. $\Phi(0)=-v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\psi\tau)\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\psi\tau)\rho(1-\alpha)(1-\tau)(\phi(\xi\psi\tau)\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}}+$

1

3. $\frac{\partial \Phi}{\partial \tilde{g}} > 0$ for $\tilde{g} > 0$

4. $\lim_{\tilde{g}\to\infty} \Phi(\tilde{g}) = \infty$

Under 1-4 there exists a unique fixed point $\tilde{g} > 0$ that solves $\Phi(\tilde{g})$, iff $\Phi(0) < 0$ which implies $v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\psi\tau)\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\psi\tau)\rho(1-\alpha)(1-\tau)(\phi(\xi\psi\tau)\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1$, for any parameter value in its assumed domain and some values of the policy instruments. Figure 1 represents the existence of positive growth rate.

Appendix 2. *Proof of Corollary 1*

From Proposition 1, for the economy to attain sustainable growth the following necessary and sufficient condition must hold

$$v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\psi\tau)\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\psi\tau)\rho(1-\alpha)(1-\tau)(\phi(\xi\psi\tau)\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1.$$

However, for $\psi = 1$, the value of the left hand side of this inequality is 0, thus $0 > 1$ implies a contradiction. Thus, Proposition 1 holds only below a maximum threshold level, $\hat{\psi}$, where $v\left((1-\hat{\psi})\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\hat{\psi}\tau)\rho+1}{(1+\phi(\xi\hat{\psi}\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\hat{\psi}\tau)\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\hat{\psi}\tau)\rho(1-\alpha)(1-\tau)(\phi(\xi\hat{\psi}\tau)\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1$ given other parameter values. In the same vein, for $\psi = 0$ Proposition 1 implies $0 > 1 \Rightarrow$ which does not hold. Thus, Proposition 1 holds above a minimum threshold level $\check{\psi}$, where $v\left((1-\check{\psi})\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\check{\psi}\tau)\rho+1}{(1+\phi(\xi\check{\psi}\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\check{\psi}\tau)\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\check{\psi}\tau)\rho(1-\alpha)(1-\tau)(\phi(\xi\check{\psi}\tau)\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1.$

Appendix 3. *Proof of Corollary 2*

From Proposition 1, for the economy to attain sustainable growth the following necessary and sufficient condition must hold:

$$v\left((1-\psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\tau)\rho+1}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\psi\tau)\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\psi\tau)\rho(1-\alpha)(1-\tau)(\phi(\xi\psi\tau)\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1.$$

However, for $\tau = 1$ the necessary and sufficient condition does not hold, for any other parameter value, because $0 > 1$ implies a contradiction. Thus, Proposition 1 holds only below a maximum threshold level $\bar{\tau}$ where $v\left((1-\psi)\bar{\tau}\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\bar{\tau})\rho+1}{(1+\phi(\xi\psi\bar{\tau})\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\psi\bar{\tau})\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\psi\bar{\tau})\rho(1-\alpha)(1-\bar{\tau})(\phi(\xi\psi\bar{\tau})\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1$ given other parameter values. In the same vein, Proposition 1 implies for $\tau = 0$ that $0 > 1$

which is a contradiction because the necessary condition does not hold. Thus, Proposition 1 holds above a minimum threshold level $\underline{\tau}$, at which $v\left((1-\psi)\underline{\tau}\right)^{1-\mu}\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}\left(\frac{\phi(\xi\psi\underline{\tau})\rho+1}{(1+\phi(\xi\psi\underline{\tau})\rho+\eta)}\right)^{(1-\alpha)(1-\mu)}\left(\frac{(1+\phi(\xi\psi\underline{\tau})\rho+\eta)^{-\alpha}(1-\mu)\eta}{\phi(\xi\psi\underline{\tau})\rho(1-\alpha)(1-\underline{\tau})(\phi(\xi\psi\underline{\tau})\rho+1)^{-\alpha}\sigma}\right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1$. Thus, when Proposition 1 holds, it holds in a range of tax rate with minimum, $\underline{\tau}$, and maximum, $\bar{\tau}$, threshold levels of income tax rate.

Figure 1 give provides a numerical respresentation for the bound of tax rate and allocation of tax revenues to health and education that guarantee positive growth.

Appendix 4. *Proof of Proposition 2.*

Solving (26) for z as a function of policy instruments we obtain

$$z=\left(\frac{v\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}((1-\psi)\tau)^{1-\mu}}{\phi(\xi\psi\tau)\left(\frac{\rho(1-\alpha)(1-\tau)\sigma}{(1-\mu)\eta}\right)\left(\frac{(\phi(\xi\psi\tau)\rho+1)}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{\mu(1-\alpha)-1}}\right)^{\frac{1}{1-\alpha\mu}} \tag{A1}$$

. Substituting (A1) to (25) we obtain the growth rate as a function of the policy instruments given by

$$\max_{\tau,\psi}\tilde{g}(\tau,\psi)=\Psi(\tau,\psi)(\frac{v\left(\frac{\mu\sigma}{(1-\mu)}\right)^{\mu}((1-\psi)\tau)^{1-\mu}}{\phi(\xi\psi\tau)\left(\frac{\rho(1-\alpha)(1-\tau)\sigma}{(1-\mu)\eta}\right)\left(\frac{(\phi(\xi\psi\tau)\rho+1)}{(1+\phi(\xi\psi\tau)\rho+\eta)}\right)^{\mu(1-\alpha)-1}})^{\frac{(1-\alpha)}{1-\alpha\mu}}-1 \tag{A2}$$

The growth maximizating conditions are given by:

$$\frac{\partial\tilde{g}(\tau,\psi)}{\partial\tau}=0,\frac{\partial\tilde{g}(\tau,\psi)}{\partial\psi}=0$$

where by solving this system of equations, after some algebra (see detailed derivations in the Companion Appendix) we obtain the policy rules (27) and (28), which together with (29) provide the growth maximizing allocations of τ^*, ψ^*, z^* as a function of model parameters.

Then, it is straightforward to prove that the growth maximizing tax rate, τ^*, is higher than the Barro (1990) optimal taxation rule, $\tau_b = 1-a$ for any positive value of ψ. In particular, $\tau^*=\frac{1-a}{1-a\psi^*}>(1-a)\Rightarrow\frac{1}{1-a\psi^*}>1\Rightarrow 1>1-a\psi^*\Rightarrow a\psi^*>0$ and because $a\in(0,1)$ then it requires $\psi^*>0$ which holds for a positive growth rate as given by Proposition 1 and Corollary 1. Also, the tax rate depends positively on health expenditures as a share of total governement revenues. In detail, the partial derivative of the growth maximizing tax rate with respect to ψ^* is positive, $\frac{\partial \tau^*}{\partial \psi^*} = \frac{(1-a)a}{(1-a\psi^*)^2} > 0$, which implies that the growth maximizing tax rate depends positively on health expenditures as a share of total government revenues.

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Table A. Values for the parameters

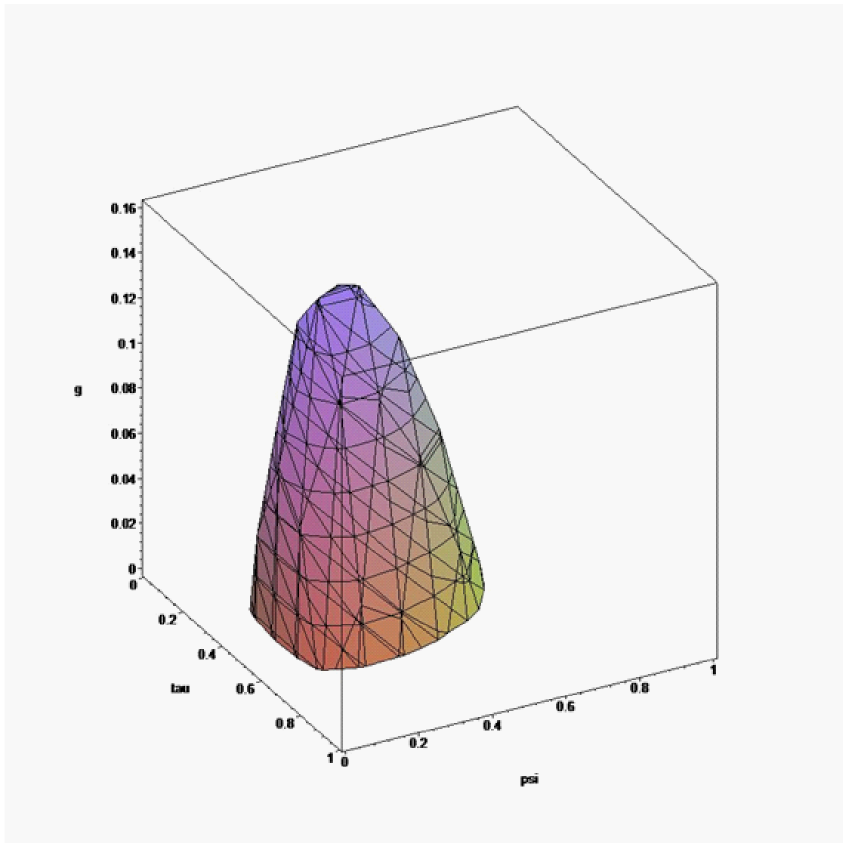
Parameter	Description	Value
α	share of physical capital in the aggregate production function	0.3
σ	cost of per child rearing time	0.1
μ	productivity parameter on private human capital	0.5
η	preference for children in the utility function	0.6
ρ	rate of time preference	0.95
v	scale parameter in human capital accumulation	10
ξ	technology of health stock production	$6-7$

Table 1 . Changes in ξ, aging and the growth-maximizing allocation

ξ	ψ^*	τ^*	z^*	g^*	n^*	ϕ^*
6.0	0.202	0.7450	154.9	0.014	1.463	0.474
6.2	0.200	0.7446	152.6	0.017	1.459	0.480
6.4	0.198	0.7442	150.5	0.020	1.455	0.485
6.6	0.197	0.7438	148.5	0.024	1.451	0.491
6.8	0.195	0.7435	146.6	0.027	1.448	0.496
7.0	0.194	0.7431	144.8	0.030	1.444	0.501

Notes: $a = 0.3$, $v = 10$, $\rho = 0.95$, $\eta = 0.6$, $\mu = 0.5$,$\sigma = 0.1$.

Figure 1. Existence of BGP and Policy Instruments



Notes: $a = 0.3$, $\upsilon = 12$, $\rho = 0.96$, $\eta = 0.6, \mu = 0.5$,$\sigma = 0.1$.